Econometrics and Practice : Mind the Gap !

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Outline

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A Jungle of Models

- There is a large family of parametric models to describe typical stylized effects of financial time series. Some stylized facts are : long memory, fat tail distribution, volatility clustering and autocorrelations of squared returns and absolute returns.
- Bollerslev [2008] has compiled a "Glossary of ARCH (GARCH)" models with more than 150 entries.

Mean equation

The mean equation of an univariate time series x_t can be described by the process

$$x_t \ = \ \mathrm{E}(x_t|\mathcal{F}_{t-1}) + \varepsilon_t \ ,$$

where $\mathrm{E}(\cdot|\cdot)$ denotes the conditional expectation operator, $\mathcal{F}_{t-1}$ the information set at time $t-1$, and ε_t the innovations of the time series.

GARCH variance equation

The mean equation does not take into account heteroskedastic effects typically observed in financial time series. Engle [1982] introduced the *Autoregressive Conditional Heteroskedastic* model, named ARCH, later generalized by Bollerslev [1986], named GARCH.

$$\begin{aligned} \varepsilon_t &= z_t \sigma_t \ , \\ z_t &\sim \mathcal{D}_\vartheta(0,1) \ , \\ \sigma_t^2 &= \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \ , \end{aligned} \tag{1}$$

Log-likelihood function

The quasi-maximum likelihood technique applied to a GARCH(1,1) process leads to the following optimization problem

$$\min \ \mathcal{L}_N(\theta) = \frac{1}{2}\sum_t \left[\ln 2\pi + \ln \sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2} \right]$$

subject to

$$x_t - \mu - \varepsilon_t = 0$$
$$\sigma_t^2 - \omega - \alpha\varepsilon_{t-1}^2 - \beta\sigma_{t-1}^2 = 0$$
$$-\omega \leq 0$$
$$-\alpha \leq 0$$
$$-\beta \leq 0$$
$$\alpha + \beta - 1 \leq 0$$

Simple R script to fit a GARCH(1,1)

First we define the negative log-likelihood function.

```
> llhGarch11 <- function(par, x) {
     mu <- par[1]; omega <- par[2]; alpha <- par[3]; beta <- par[4]
     e2 <- (x - mu)^2
     e2t <- omega + alpha*c(mean(e2),e2[-length(x)])
     s2 <- filter(e2t, beta, "recursive", init = mean(e2))
     0.5*sum( log(2*pi) + log(s2) + e2/s2)
 }
```

Simple R script to fit a GARCH(1,1)

Second we minimize the negative log-likelihood function and use typical values for the initial parameters.

```
> x <- dem2gbp[,1]
> mu <- mean(x); omega <- 0.1*var(x); alpha <- 0.1; beta <- 0.8
> par <- c(mu, omega, alpha, beta)
> low <- c(-10*abs(mu), 0, 0, 0)
> up <- c(10*abs(mu), 100*abs(mu), 1, 1)
> fit <- nlminb(star=par, objective=llhGarch11, x = x, lower=low, upper=up)$par
> names(fit) <- c("mu", "omega", "alpha", "beta")
> round(fit, 5)
```

```
      mu    omega    alpha     beta
-0.00619  0.01076  0.15313  0.80597
```

The Art of Estimating Parameters

- Brooks [2001] compared different implimentation of GARCH model and obtained very heterogenous results.
- Numerical errors can come from : misspecifications, different parameter initializations, different values for ε_0 and σ_0 and different optimization schemes.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

- Zivot [2008] reviewed practical issues encoutered while estimating GARCH models.

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Econometrics Models

- A model is a mental representation which captures the essence of a process.
- The phenomena is described by a set of rules, pictures, methods which break the process into simple concepts.
- This means that data sets might contain more information than what can be described by one model.
- Econometricians usually theorize about different characteristic features, for example stylized facts, rather than on a particular market (Stigum [2003]).

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Stationarity

The goal of parametric models usually is to describe a conditional density function of $y_1, ..., y_T$ which can be decomposed as

$$g(y_1, ..., y_T | y_o) = \prod_{t=1}^{T} g_t(y_t | x_t) = \prod_{t=1}^{T} g(y_t | x_t), \tag{2}$$

under a strict stationarity assumption on Y.

Stationarity

For a given time series, Y_t, we say that the process is

- strict stationary if the joint distribution of $(y_{t_1}, ..., y_{t_k})$ is the same as of $(y_{t_1+h}, ..., y_{t_k+h}) \quad \forall h$
- or weakly stationary (covariance stationary) when

$$\begin{aligned} E[y_t] &= \mu, \quad \forall t \\ Cov(y_{t+h}, y_t) &= Cov(y_{t+h+j}, y_{t+1}), \quad \forall j \quad and \quad \forall h. \end{aligned}$$

Stationarity

- Although many models assume that observations are at least covariance stationary, it has been shown that data is often non-stationary. (Garcia and Perron [1996], Bai and Perron [1998], [1998], Timmermann [2001])
- It has been also shown that a lack of covariance stationarity might be caused of market crashes (Dehay [1996]).

Outliers

- Outliers can be viewed as part of the data generated by a process which is not reflected by the model used.
- Fox [1972] and Abraham and Box [1979] introduced two characterizations of outliers in finance : additive or innovation outliers.

Outliers

Additive (AO) and innovative (IO) outliers at time k can be defined as

- AO :

$$y_t = E(x_t|x_{t-1}) + \Delta_1 \delta_{t,k} + \varepsilon_t$$

- IO :

$$y_t = E(y_t|y_{t-1}) + \Delta_2 \delta_{t,k} + \varepsilon_t$$

where $x_t = E(x_t|x_{t-1}) + \varepsilon_t$ is a stationary time series, y_t denotes the observed time series, ε_t are the innovations, Δ the magnitude of the shift and δ the Kronecker symbol ($\delta_{t,k} = 1 \text{ for } t = k$, $\delta_{t,k} = 0 \text{ for } t \neq k$).

Outliers

- The outliers might have a significant impact on the results of the standard methods.
- From a statistical point of view it would be “recommended” to use robust methods.
- But in practice outliers really matter !
- For example robust estimation of the covariance matrix in portfolio optimization can lead to underestimated risk.

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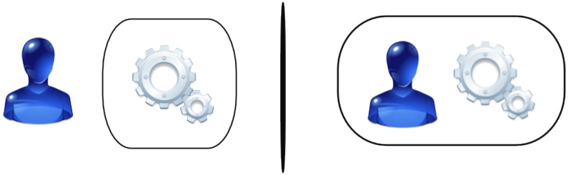
Paradigm
Observer observed

- In natural science, the observer is not part of the phenomena studied.
- But in financial applications the observer is part of the process because he adds his perception (knowledge) to the process.



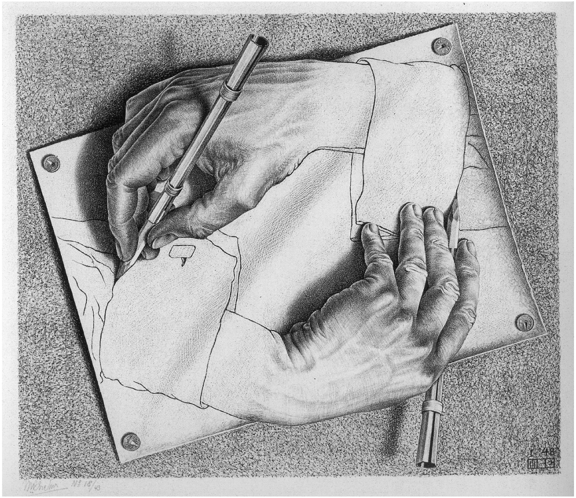


Figure: Drawing Hands by M. C. Escher, 1948, Lithograph Scanned from The Magic of M. C. Escher. (Harry N. Abrams, Inc. ISBN 0-8109-6720-0)

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Regime models

- Rapid changes in market conditions seem to characterize many models in financial applications.
- Major changes could arise from factors such as rumors, news, speculative bubbles and new policies.
- Regime could be one approach in order to combine current models with data encountered in practice. (Pesaran and Timmermann [2004], Guidolin and Timmermann [2007])

Markov Switching Models

- Markov Switching models have been introduced to include abrupt parameter changes in models. The idea is to add a hidden state variable which allows changing the parameter when the process is in a different state.
- Caporale et al. [2003] showed that high persistence in GARCH models can be the consequence of a structural break. He showed that one can often decompose an IGARCH process to a MS-GARCH.

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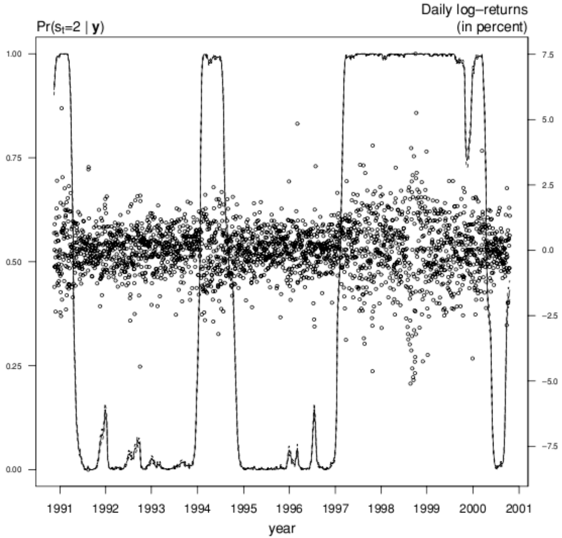
Bayesian MS-GARCH

The transition probability plot of a Bayesian MS-GARCH model with Student-t innovations applied on the Swiss Market Index (Ardia [2009]).



Indirect Inference

- Indirect Inference is a simulation based method for estimating the parameters of model with an auxiliary model.
- The goal is to choose the parameters of the model which reproduce the data set as closely as possible.

Indirect Inference

For a model defined with the conditional density function

$$g(y_t|x_t;\rho)$$

where x_t is the finite history of Y up to time $t-1$.
The auxiliary model can be specified by

$$f(y_t|x_t;\theta)$$

and its parameters θ can be estimated using the observed data by maximizing the likelihood function

$$\hat{\theta}_T = \arg\max_{\theta} \sum_t \log f(y_t|x_t,\theta)$$

Indirect Inference

Given simulated sample $(\tilde{y}_t, \tilde{x}_t) = (\tilde{y}_t(\rho), \tilde{x}_t(\rho))$ with $t = 1, ..., S$

$$\tilde{\theta}_S = \arg\max_{\theta} \frac{1}{S} \sum_t \log f(\tilde{y}_t | \tilde{x}_t, \theta)$$

The idea is to find ρ_0 by minimizing the distance between $\hat{\theta}_T$ and $\tilde{\theta}_S$

$$\hat{\rho}_{TS}(\Omega) = \arg\min_{\rho} ||\hat{\theta}_T - \tilde{\theta}_S||_{\Omega}$$

BL Portfolio Optimization

- Black and Litterman introduced in 1990 a portfolio optimization framework where equilibrium is combined with investor-specific views.
- We know from behavioral finance that investors tend to be overconfident when estimating their confidence intervals. This can be problematic in the BL framework (Mankert [2006]).

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```
> toLatex(sessionInfo())
```

- R version 2.10.0 Under development (unstable) (2009-04-19 r48357), `i686-pc-linux-gnu`
- Locale: `LC_CTYPE=en_US.UTF-8;LC_NUMERI ...`
- Base packages: base, datasets, graphics, grDevices, methods, stats, utils
- Other packages: fBasics 2100.77, fGarch 2100.78, MASS 7.2-46, timeDate 290.85, timeSeries 2100.83

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