Using R for Hedge Fund of Funds Risk Management

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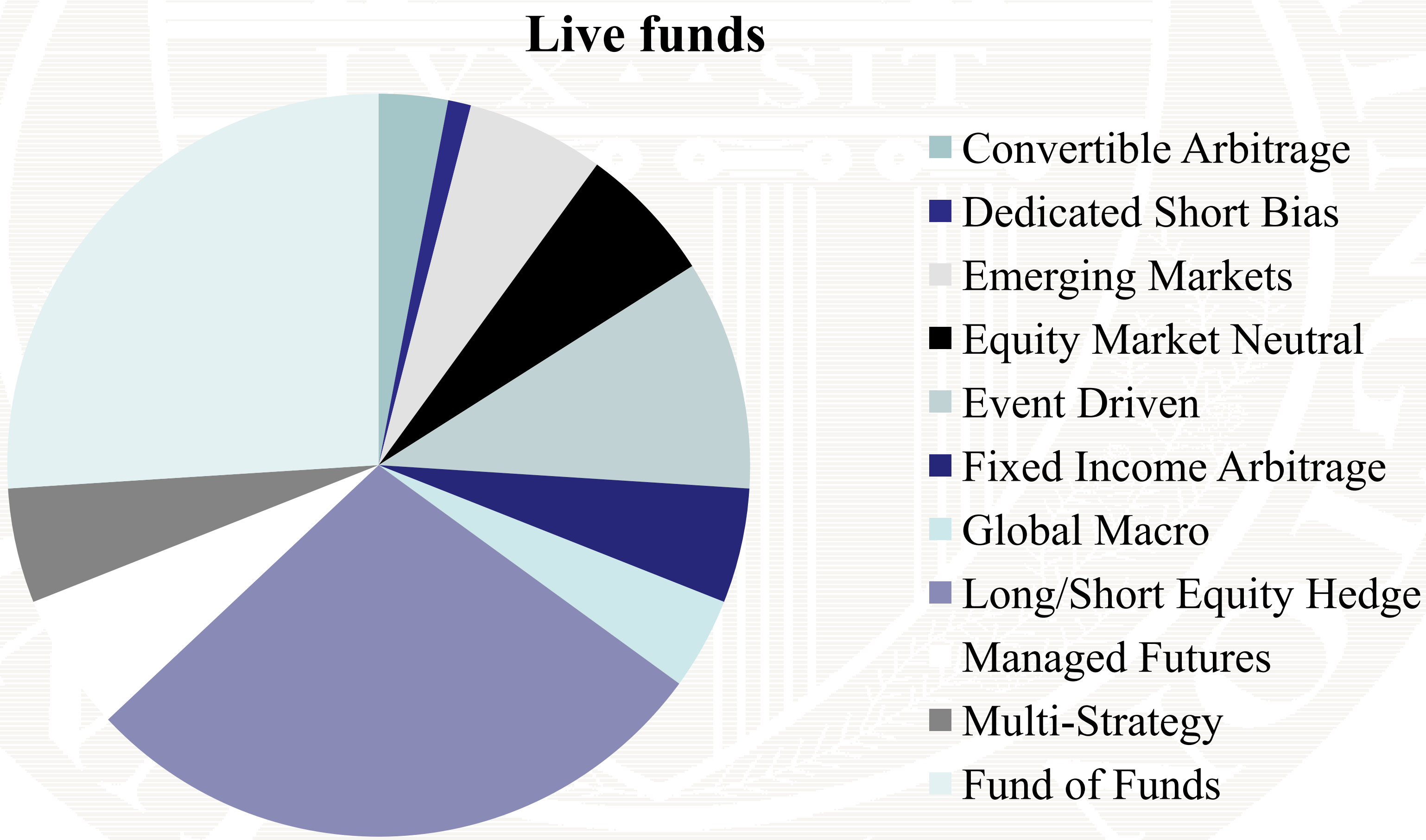
Outline

- Hedge fund of funds environment
- Factor model risk measurement
- R implementation in corporate environment
- Dealing with unequal data histories
- Some thoughts on S-PLUS and S+FinMetrics vs. R in Finance

Hedge Fund of Funds Environment

- HFoFs are hedge funds that invest in other hedge funds
  - 20 to 30 portfolios of hedge funds
  - Typical portfolio size is 30 funds
- Hedge fund universe is large: 5000 live funds
  - Segmented into 10-15 distinct strategy types
- Hedge funds voluntarily report monthly performance to commercial databases
  - Altvest, CISDM, HedgeFund.net, Lipper TASS, CS/Tremont, HFR
- HFoFs often have partial position level data on invested funds

Hedge Fund Universe



Characteristics of Monthly Returns

- Reporting biases
  - Survivorship, backfill
- Non-normal behavior
  - Asymmetry (skewness) and fat tails (excess kurtosis)
- Serial correlation
  - Performance smoothing, illiquid positions
- Unequal histories

Characteristics of Hedge Fund Data

	fund1	fund2	fund3	fund4	fund5
Observations	122.0000	107.0000	135.0000	135.0000	135.0000
NAs	13.0000	28.0000	0.0000	0.0000	0.0000
Minimum	-0.0842	-0.3649	-0.0519	-0.1556	-0.2900
Quartile 1	-0.0016	-0.0051	0.0020	-0.0017	-0.0021
Median	0.0058	0.0046	0.0060	0.0073	0.0049
Arithmetic Mean	0.0038	-0.0017	0.0063	0.0059	0.0021
Geometric Mean	0.0037	-0.0029	0.0062	0.0055	0.0014
Quartile 3	0.0158	0.0129	0.0127	0.0157	0.0127
Maximum	0.0311	0.0861	0.0502	0.0762	0.0877
Variance	0.0003	0.0020	0.0002	0.0008	0.0013
Stdev	0.0176	0.0443	0.0152	0.0275	0.0357
Skewness	-1.7753	-5.6202	-0.8810	-2.4839	-4.9948
Kurtosis	5.2887	40.9681	3.7960	13.8201	35.8623
Rho1	0.6060	0.3820	0.3590	0.4400	0.383

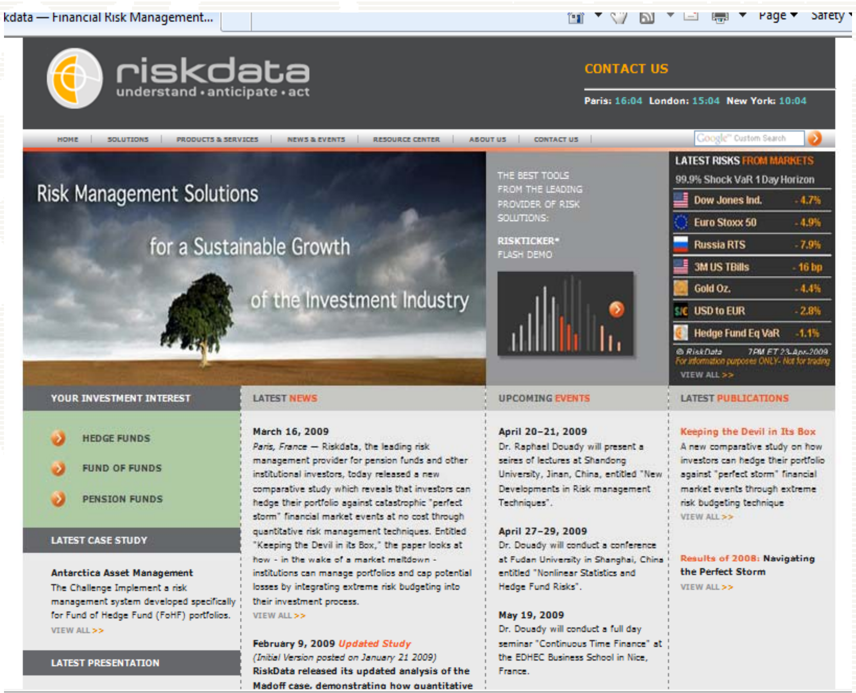
Sample: January 1998 – March 2009

Factor Model Risk Measurement in HFoFs Portfolio

- Quantify factor risk exposures
  - Equity, rates, credit, volatility, currency, commodity, etc.
- Quantify tail risk
  - VaR, ETL
- Risk budgeting
  - Component, incremental, marginal
- Stress testing and scenario analysis

Commercial Products

www.riskdata.com



www.finanalytica.com

Very expensive! R is not!

Factor Model: Methodology

$$R_{it} = \alpha_i + \beta_{i1}F_{1t} + \cdots + \beta_{ik}F_{kt} + \varepsilon_{it},$$

$$= \alpha_i + \boldsymbol{\beta}_i'\mathbf{F}_\mathbf{t} + \varepsilon_{it}$$

$$i = 1, \ldots, n;\; t = t_i, \ldots, T$$

$$\mathbf{F}_t \sim \; (\boldsymbol{\mu}_\mathrm{F}, \boldsymbol{\Sigma}_F)$$

$$\varepsilon_{it} \sim \; (0, \sigma^2_{\varepsilon,i})$$

$$\mathrm{cov}(f_{jt}, \varepsilon_{it}) = 0 \text{ for all } j,\; i \text{ and } t$$

$$\mathrm{cov}(\varepsilon_{it}, \varepsilon_{jt}) = 0 \text{ for } i \neq j$$

Practical Considerations

- Many potential risk factors (> 50)
- High collinearity among some factors
- Risk factors vary across discipline/strategy
- Nonlinear effects
- Dynamic effects
- Time varying coefficients
- Common histories for factors; unequal histories for fund performance

Expected Return Decomposition

$$E[R_{it}] = \alpha_i + \beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}]$$

Expected return due to “beta” exposure

$$\beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}]$$

Expected return due to manager specific “alpha”

$$\alpha_i = E[R_{it}] - (\beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}])$$

Variance Decomposition

$$\text{var}(R_{it}) = \underbrace{\boldsymbol{\beta}_i' \,\text{var}(\mathbf{F}_t)\boldsymbol{\beta}_i}_{\text{systematic}} + \underbrace{\text{var}(\varepsilon_{it})}_{\text{specific}} = \boldsymbol{\beta}_i'\boldsymbol{\Sigma}_{\mathbf{F}}\boldsymbol{\beta}_i + \sigma_{\varepsilon,i}^2$$

Variance contribution due to factor exposures

$$\beta_1^2 \,\text{var}(F_{1t}) + \beta_2^2 \,\text{var}(F_{2t}) + \cdots + \beta_k^2 \,\text{var}(F_{kt})$$

Variance contribution due to covariances between factors

$$2\beta_1\beta_2 \,\text{cov}(F_{1t}, F_{2t}) + \cdots + 2\beta_{k-1}\beta_k \,\text{cov}(F_{k-1t}, F_{kt})$$

Covariance

$$\underset{n\times 1}{\boldsymbol{R}_t} = \underset{n\times 1}{\boldsymbol{\alpha}} + \underset{n\times k}{\boldsymbol{B}}\,\underset{k\times 1}{\boldsymbol{F}_t} + \underset{n\times 1}{\boldsymbol{\varepsilon}_t}$$

$$\mathrm{var}(R_t) = \Sigma_{FM} = \mathbf{B}\boldsymbol{\Sigma}_{\mathbf{F}}\mathbf{B}' + \mathbf{D}_\varepsilon$$

$$\mathbf{D}_\varepsilon = diag(\sigma^2_{\varepsilon,1},\ldots,\sigma^2_{\varepsilon,n})$$

Note: $\mathrm{cov}(R_{it}, R_{jt}) = \boldsymbol{\beta}_i'\,\mathrm{var}(\mathbf{F}_t)\boldsymbol{\beta}_j = \boldsymbol{\beta}_i'\boldsymbol{\Sigma}_{\mathbf{F}}\boldsymbol{\beta}_j$

Portfolio Analysis

$\mathbf{w} = (w_1, \ldots, w_n)' =$ portfolio weights

$$
\begin{aligned}
R_{pt} &= \mathbf{w}'\mathbf{R}_t = \mathbf{w}'\boldsymbol{\alpha} + \mathbf{w}'\mathbf{B}\mathbf{F}_t + \mathbf{w}'\boldsymbol{\varepsilon}_t \\
&= \sum_{i=1}^{n} w_i R_{it} = \sum_{i=1}^{n} w_i \alpha_i + \sum_{i=1}^{n} w_i \boldsymbol{\beta}_i' \mathbf{F}_t + \sum_{i=1}^{n} w_i \varepsilon_{it} \\
&= \alpha_p + \boldsymbol{\beta}_p' \mathbf{F}_t + \varepsilon_{pt}
\end{aligned}
$$

Portfolio Variance Decomposition

$$\sigma_p^2 = \text{var}(R_{pt}) = \mathbf{w}'\,\text{var}(\mathbf{R}_t)\mathbf{w} = \mathbf{w}'\mathbf{B}\boldsymbol{\Sigma}_F\mathbf{B}'\mathbf{w} + \mathbf{w}'\mathbf{D}\mathbf{w}$$

$$\sigma_{p,systematic}^2 = \mathbf{w}'\mathbf{B}\boldsymbol{\Sigma}_F\mathbf{B}'\mathbf{w}$$

$$\sigma_{p,specific}^2 = \mathbf{w}'\mathbf{D}\mathbf{w} = \sum_{i=1}^{n} w_i \sigma_{\varepsilon,i}^2 \qquad R_p^2 = \frac{\sigma_{p,systematic}^2}{\sigma_p^2}$$

$$\sigma_{p,systematic}^2 = \beta_p' \Sigma_F \beta_p = \sum_{j=1}^{k} \beta_{p,j}^2 \sigma_{jj}^2 + \text{covariance terms}$$

$$\text{covariance terms} = \sigma_{p,systematic}^2 - \sum_{j=1}^{k} \beta_{p,j}^2 \sigma_{jj}^2$$

Risk Budgeting: Volatility

$$\mathbf{MCR} = \frac{\partial \sigma_p}{\partial \mathbf{w}} = \frac{\mathbf{B\Sigma_F B'w + Dw}}{\sigma_p}$$

Marginal contributions to risk

$$\mathbf{MCR}_{systematic} = \frac{\mathbf{B\Sigma_F B'w}}{\sigma_p}$$

$$\mathbf{MCR}_{specific} = \frac{\mathbf{Dw}}{\sigma_p}$$

$$\mathbf{CR} = \mathbf{w} \odot \frac{\partial \sigma_p}{\partial \mathbf{w}} = \frac{\mathbf{w} \odot (\mathbf{B\Sigma_F B'w + Dw})}{\sigma_p}$$

Components to risk

$$\mathbf{1'CR} = \sum_{i=1}^{n} CR_i = \sigma_p$$

Tail Risk Measures

Value-at-Risk (VaR)

$$VaR_\alpha = -q_\alpha = -F^{-1}(\alpha)$$

$$F = CDF \text{ of returns } R$$

Expected Shortfall (ES)

$$ES_\alpha = -E[R \mid R \leq VaR_\alpha]$$

Tail Risk Measures: Normal Distribution

$$R_p \sim N(\mu_p, \sigma_p^2), \; \sigma_p^2 = w'\Sigma_{FM} w$$

$$VaR_\alpha^N = -\mu_p - \sigma_p \times z_\alpha, \; z_\alpha = \Phi^{-1}(\alpha)$$

$$ES_\alpha^N = \mu_p - \sigma_p \frac{1}{\alpha}\phi(z_\alpha)$$

See functions in PerformanceAnalytics

Tail Risk Measures: Non-Normal Distributions

Use Cornish-Fisher expansion to account for asymmetry and fat tails

$$VaR_{\alpha}^{CF} = -\mu_i - \sigma_i \times z_{\alpha}$$
$$+\sigma_i \left[-\frac{1}{6}\left(z_{\alpha}^2 - 1\right) skew_i - \frac{1}{24}\left(z_{\alpha}^3 - 3z_{\alpha}\right) ekurt_i + \frac{1}{36}\left(2z_{\alpha}^3 - 5z_{\alpha}\right) skew_i^2 \right]$$

ES_{α}^{CF} : Formula given in Boudt, Peterson and Croux (2008) "Estimation and Decomposition of Downside Risk for Portfolios with Non-Normal Returns," *Journal of Risk* and implementation in PerformanceAnalytics

Risk Budgeting: Tail Risk

Value-at-Risk (VaR)

$$VaR_\alpha = \sum_{i=1}^{n} w_i \frac{\partial VaR_\alpha}{\partial w_i} = \sum_{i=1}^{n} \overbrace{w_i \times mVaR_{\alpha,i}}^{cVaR_{\alpha,i}},$$

$$mVaR_{\alpha,i} = \frac{\partial VaR_\alpha}{\partial w_i} = -E[R_i \mid R_p = VaR_\alpha]$$

Expected Shortfall (ES)

$$ES_\alpha = \sum_{i=1}^{n} w_i \frac{\partial ES_\alpha}{\partial w_i} = \sum_{i=1}^{n} \overbrace{w_i \times mES_{\alpha,i}}^{cES_{\alpha,i}},$$

$$mES_{\alpha,i} = \frac{\partial ES_\alpha}{\partial w_i} = -E[R_i \mid R_p \le VaR_\alpha]$$

Risk Budgeting: Explicit Formulas

- Normal distribution
  - See Jorian (2007) or Dowd (2002)
- Non-normal distribution using Cornish-Fisher expansion
  - See Boudt, Peterson and Croux (2008) "Estimation and Decomposition of Downside Risk for Portfolios with Non-Normal Returns," *Journal of Risk* and implementation in PerformanceAnalytics

Risk Budgeting: Simulation

$\{R_{it}\}_{t=1}^{M} = M$ simulated returns

Method 1: Brute Force

$$mVaR_{\alpha,i} \approx \frac{\Delta VaR_{\alpha}}{\Delta w_i}\ ,\ mES_{\alpha,i} = \frac{\Delta ES_{\alpha}}{\Delta w_i}$$

Method 2: Average R_{it} around values for which $R_{pt} = VaR_{\alpha}$

$$mVaR_{\alpha,i} \approx -\sum_{t:R_{pt}=VaR_{\alpha}\pm\varepsilon} R_{it}\ ,\ mES_{\alpha,i} \approx -\sum_{t:R_{pt}\le VaR_{\alpha}} R_{it}$$

R Functions for Factor Model Risk Analysis

Function	Function
factorModelCovariance	normalES
factorModelRiskDecomposition	normalPortfolioES
normalVaR	normalMarginalES
normalPortfolioVaR	normalComponentES
normalMarginalVaR	modifiedES
normalComponentVaR	modifiedPortfolioES
normalVaRreport	modifiedESreport
modifiedVaR	simulatedMarginalVaR
modifiedPortfolioVaR	simulatedComponentVaR
modifiedMarginalVaR	simulatedMarginalES
modifiedComponentVaR	simulatedComponentES

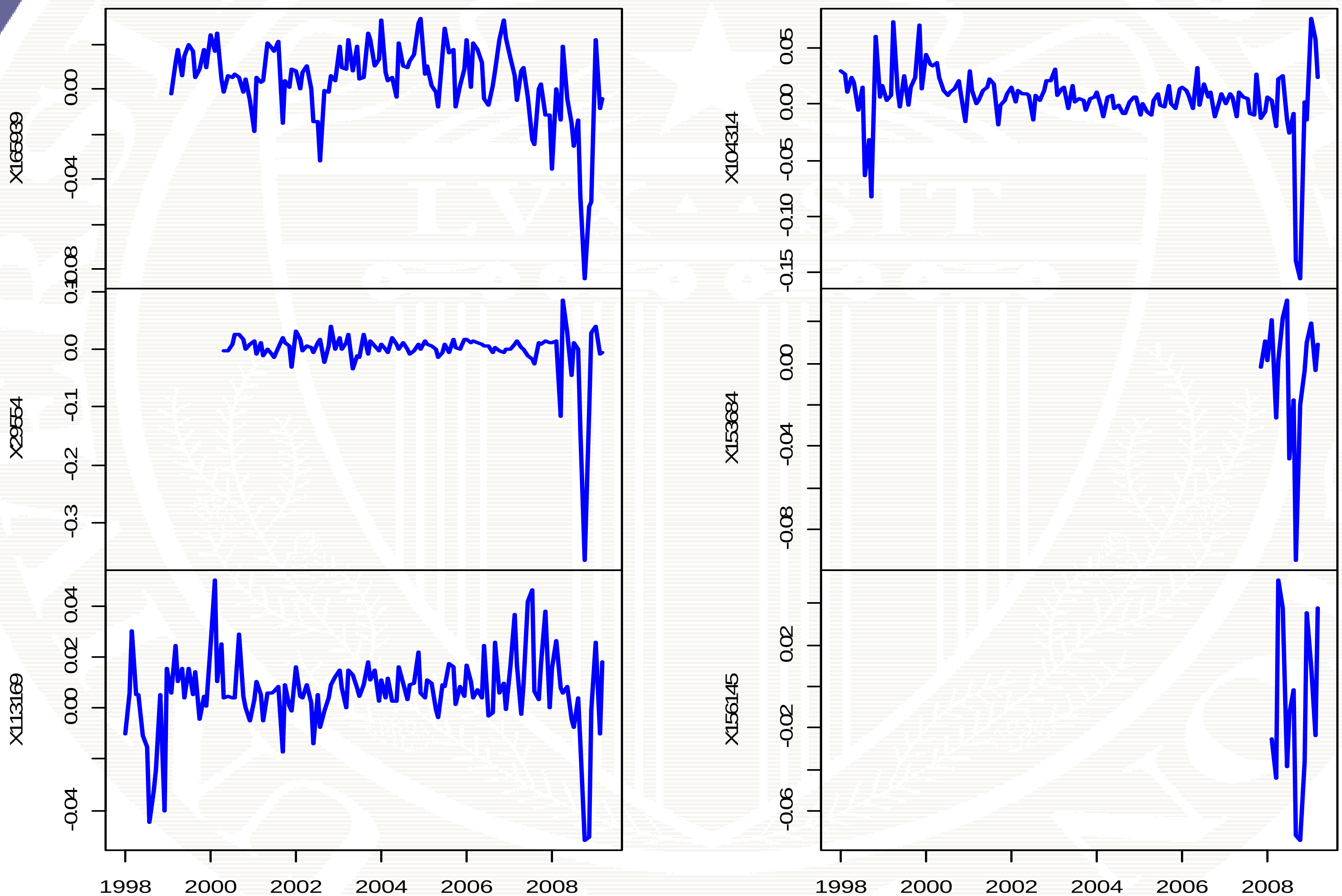
Unequal Histories

Risk factors	Fund performance	
$F_{1,T}, \ldots, F_{k,T}$	$R_{1,T}$	$R_{n,T}$
$\vdots$	$\vdots$	$\vdots$
$F_{1,T-T_i}, \ldots, F_{k,T-T_i}$	$R_{1,T-T_1}$	$\vdots$
$\vdots$		$R_{n,T-T_n}$
$F_{1,1}, \ldots, F_{k,1}$		

Observe full history

Observe partial histories

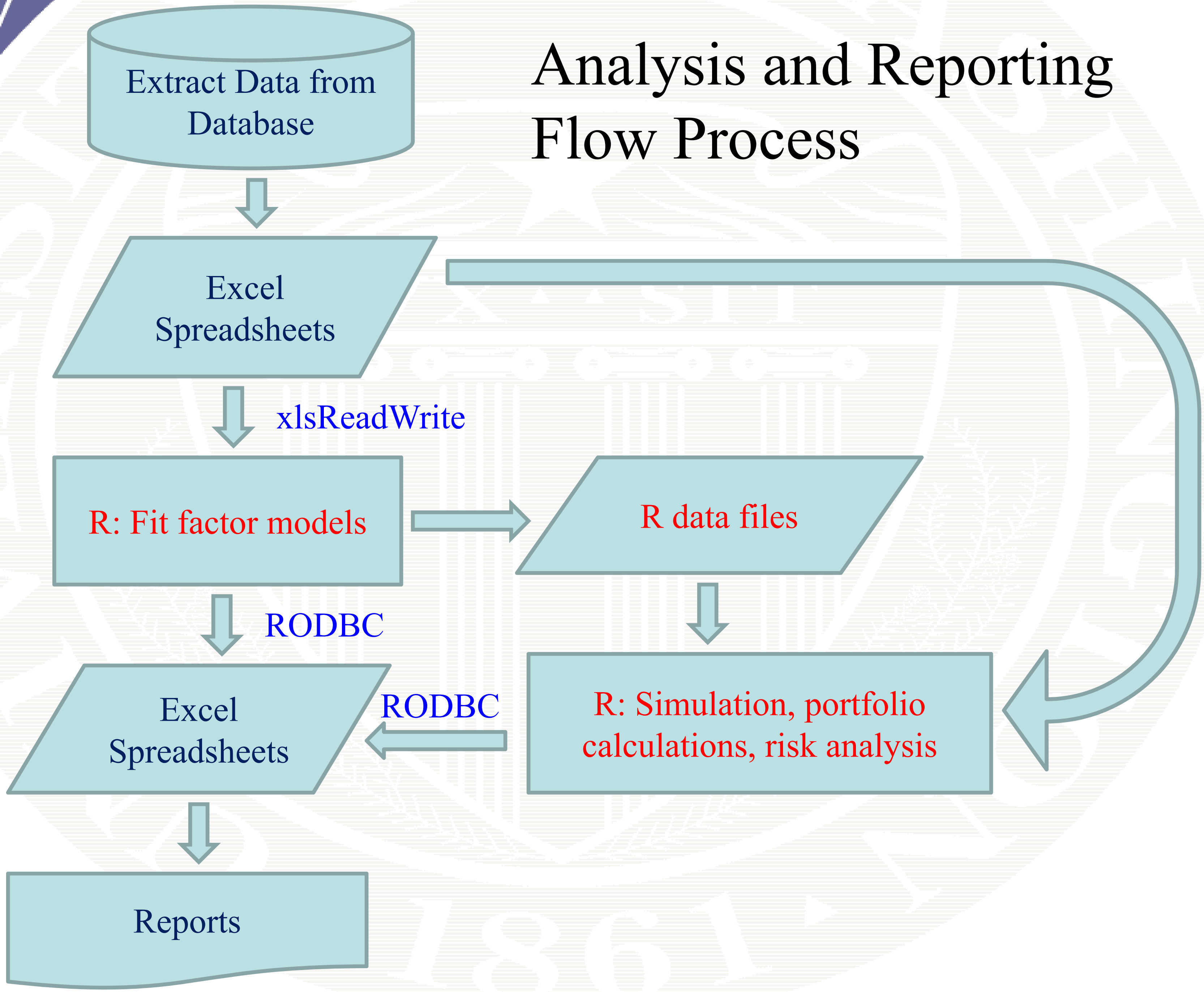
Example Portfolio: Unequal Histories of Individual Funds



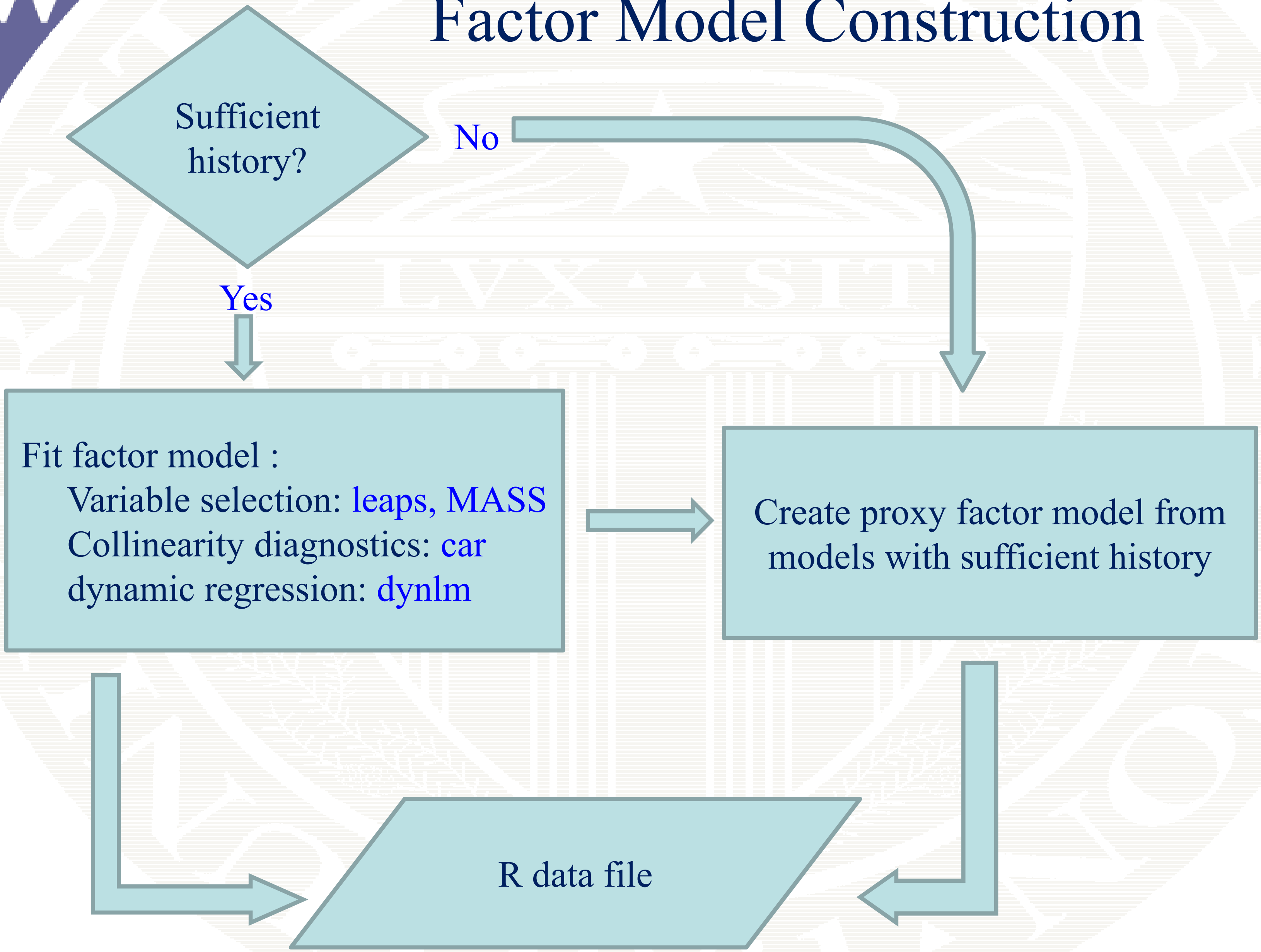
Implication of Unequal Histories

- Can’t fit factor models to some funds
  - Need to create proxy factor model
- Statistics on common histories (truncated data) may be unreliable
- Difficult to compute non-normal tail risk measures

Analysis and Reporting Flow Process



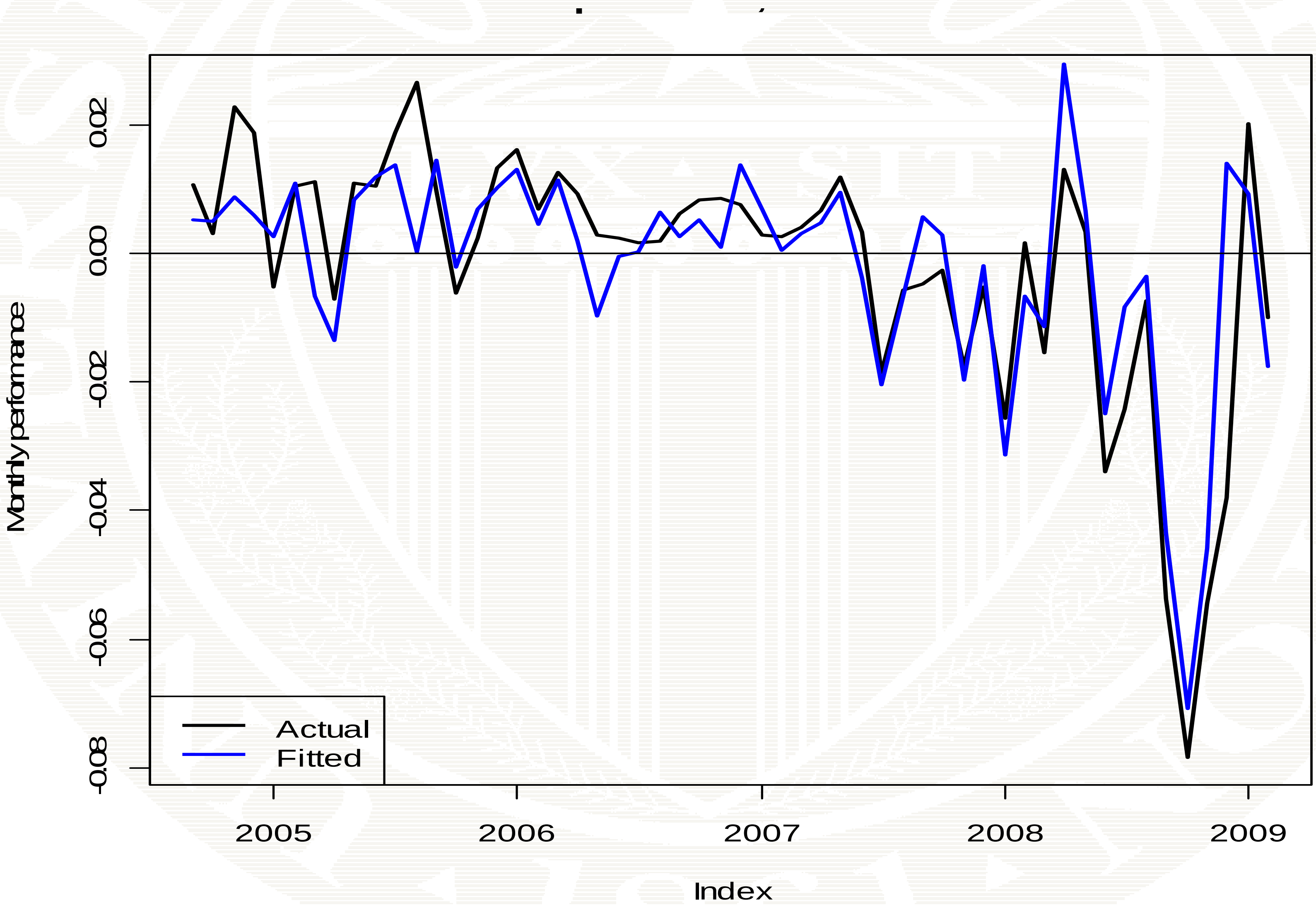
Factor Model Construction



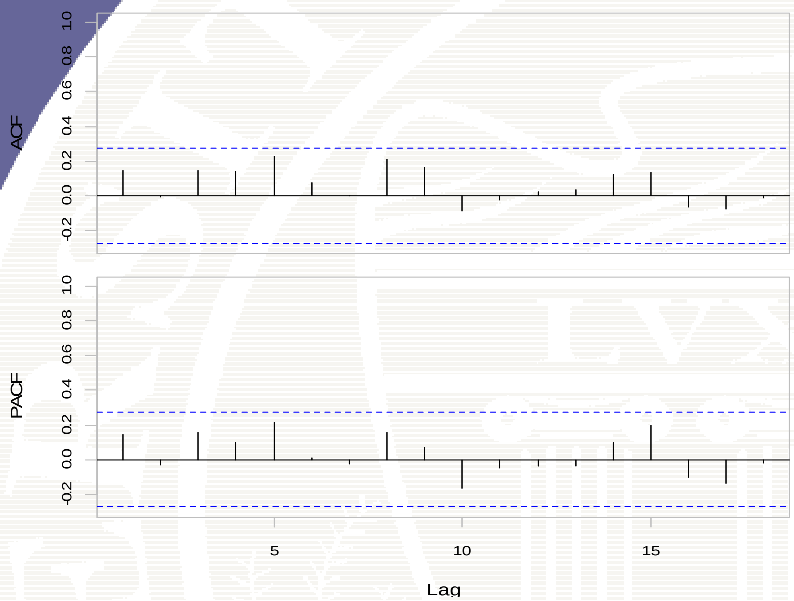
Evaluation of Fitted Factor Models

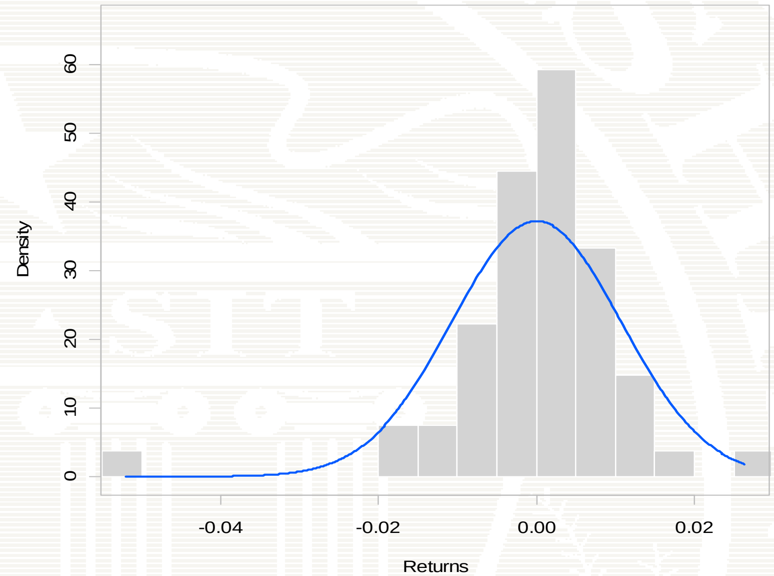
- Graphical diagnostics
  - Created plot method appropriate for time series regression.
- Stability analysis
  - CUSUM etc: strucchange
  - Rolling analysis: rollapply (zoo)
  - Time varying parameters: dlm
- Dynamic effects
  - dynlm, lmtest

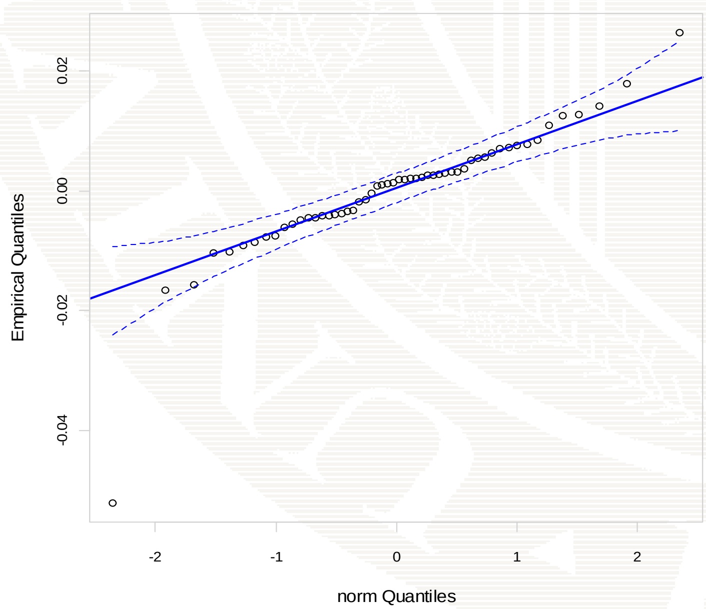
Diagnostic Plots: Example Fund

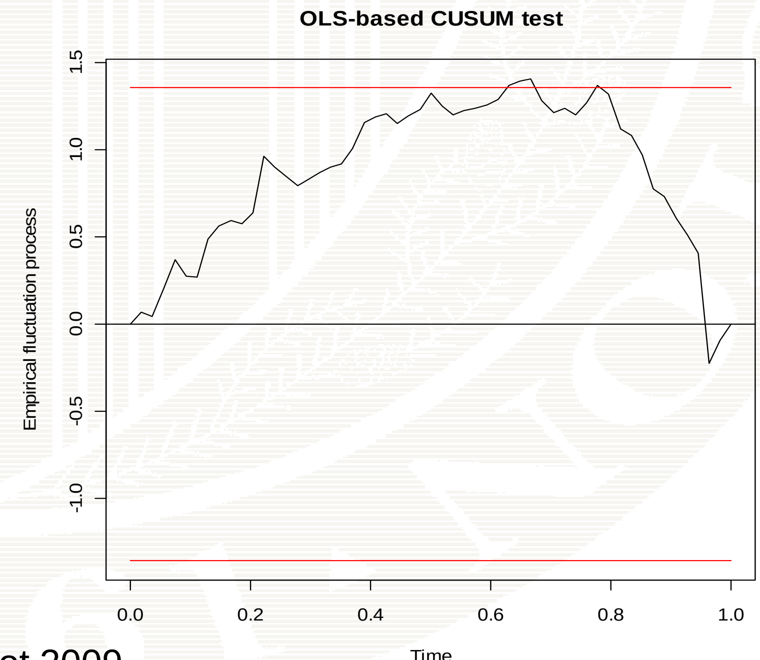


Time Series Regression Residual diagnostics

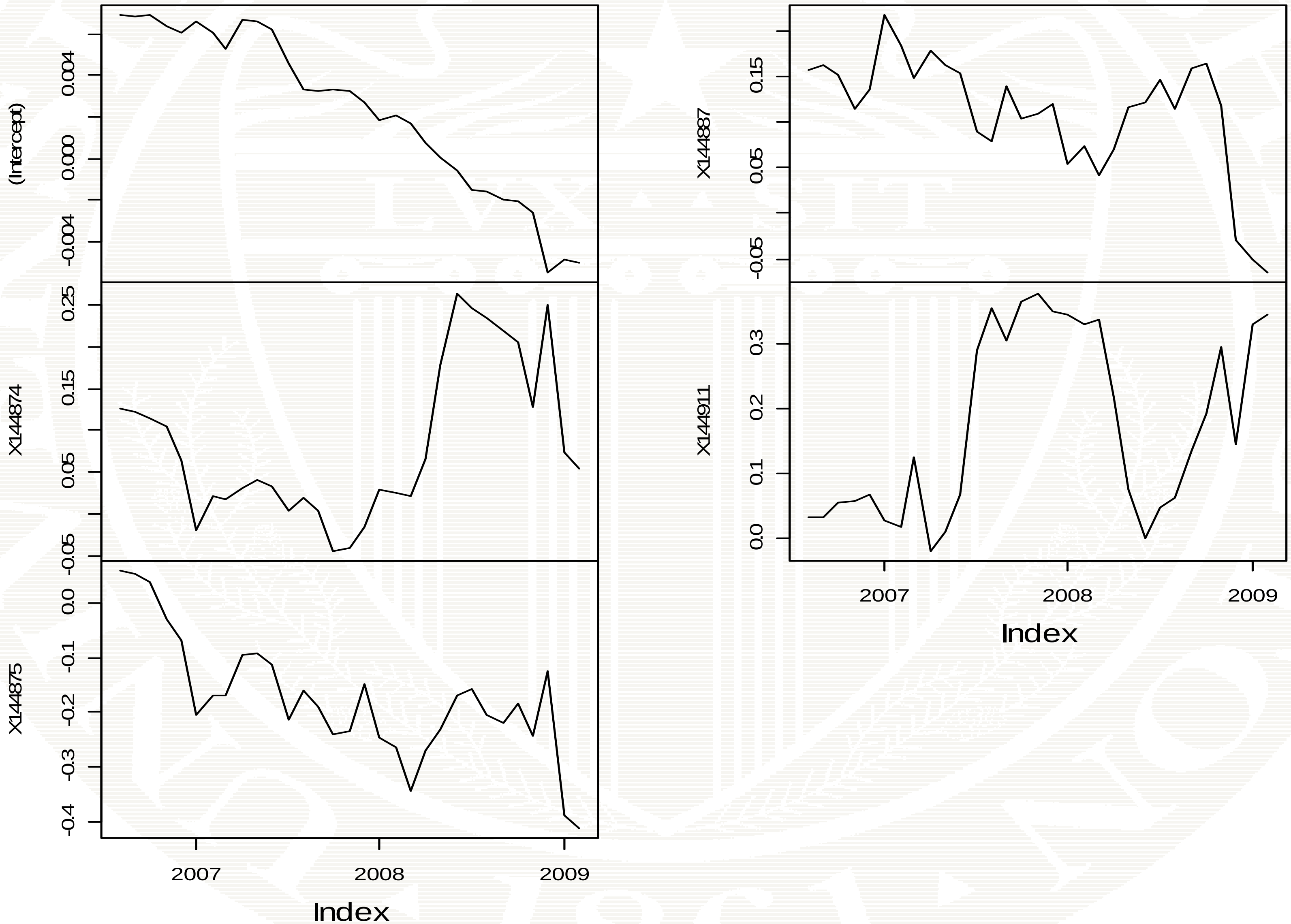








24-month rolling estimates



Dealing with Unequal Histories

- Estimate conditional distribution of R_i given $\boldsymbol{F}$
  - Fitted factor model or proxy factor model
- Estimate marginal distribution of $\boldsymbol{F}$
  - Empirical distribution, multivariate normal, copula
- Derive marginal distribution of R_i from $p(R_i|\boldsymbol{F})$ and $p(\boldsymbol{F})$
- Simulate R_i and Calculate functional of interest
  - Unobserved performance, Sharpe ratio, ETL etc

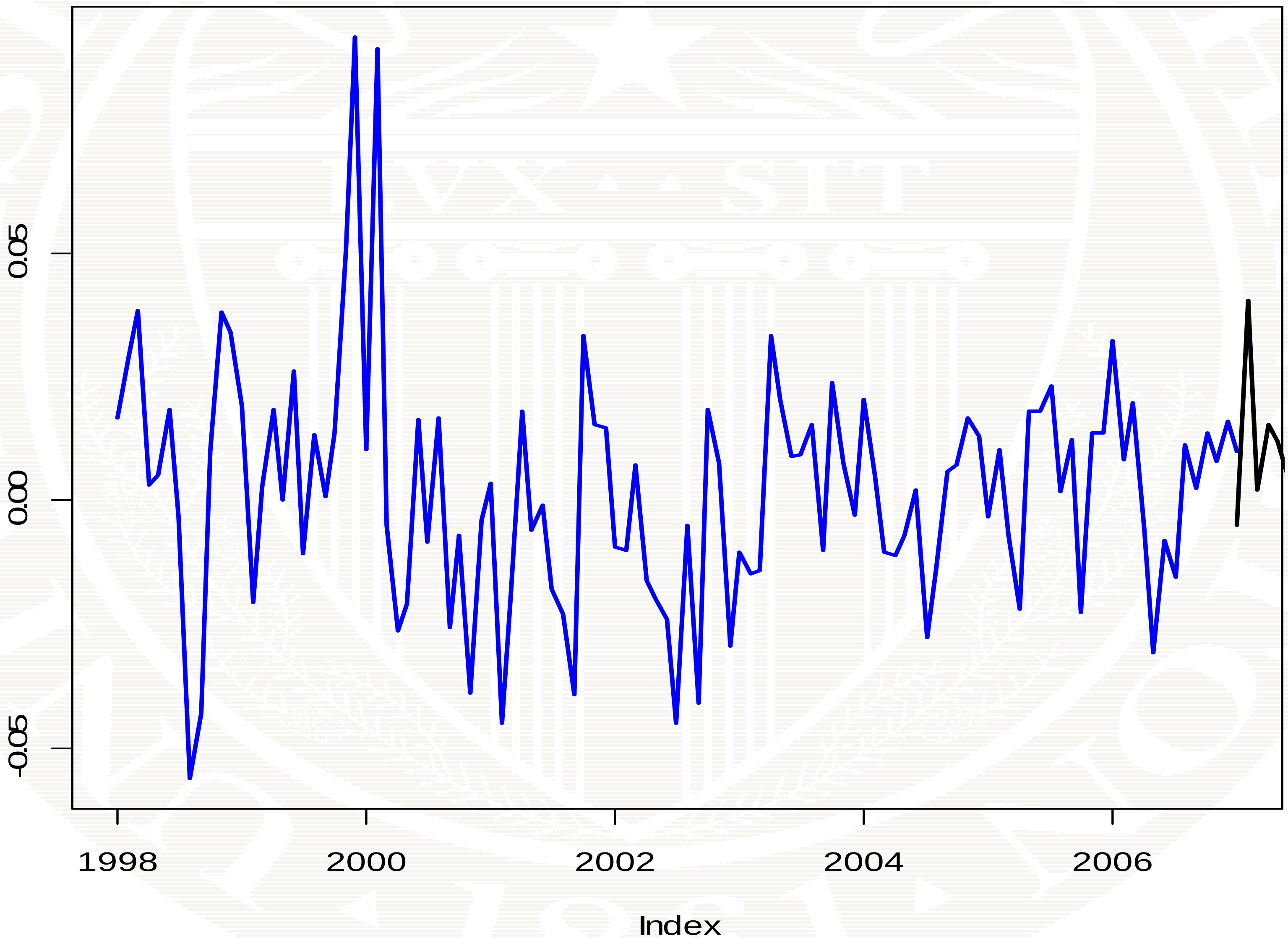
Simulation Algorithm

- Draw $\left\{ \widetilde{\boldsymbol{F}}_1, \ldots, \widetilde{\boldsymbol{F}}_M \right\}$ by resampling from the empirical distribution of ***F*.**
- For each $\widetilde{\boldsymbol{F}_u}$ ($u = 1, \ldots, M$), draw a value $\widetilde{R}_{i,u}$ from the estimated conditional distribution of R_i given $\boldsymbol{F} = \widetilde{\boldsymbol{F}}_u$ (e.g., from fitted factor model assuming normal errors)
- $\left\{ \widetilde{R}_{i,u} \right\}_{u=1}^{M}$ is the desired sample for R_i
- $M \approx 5000$

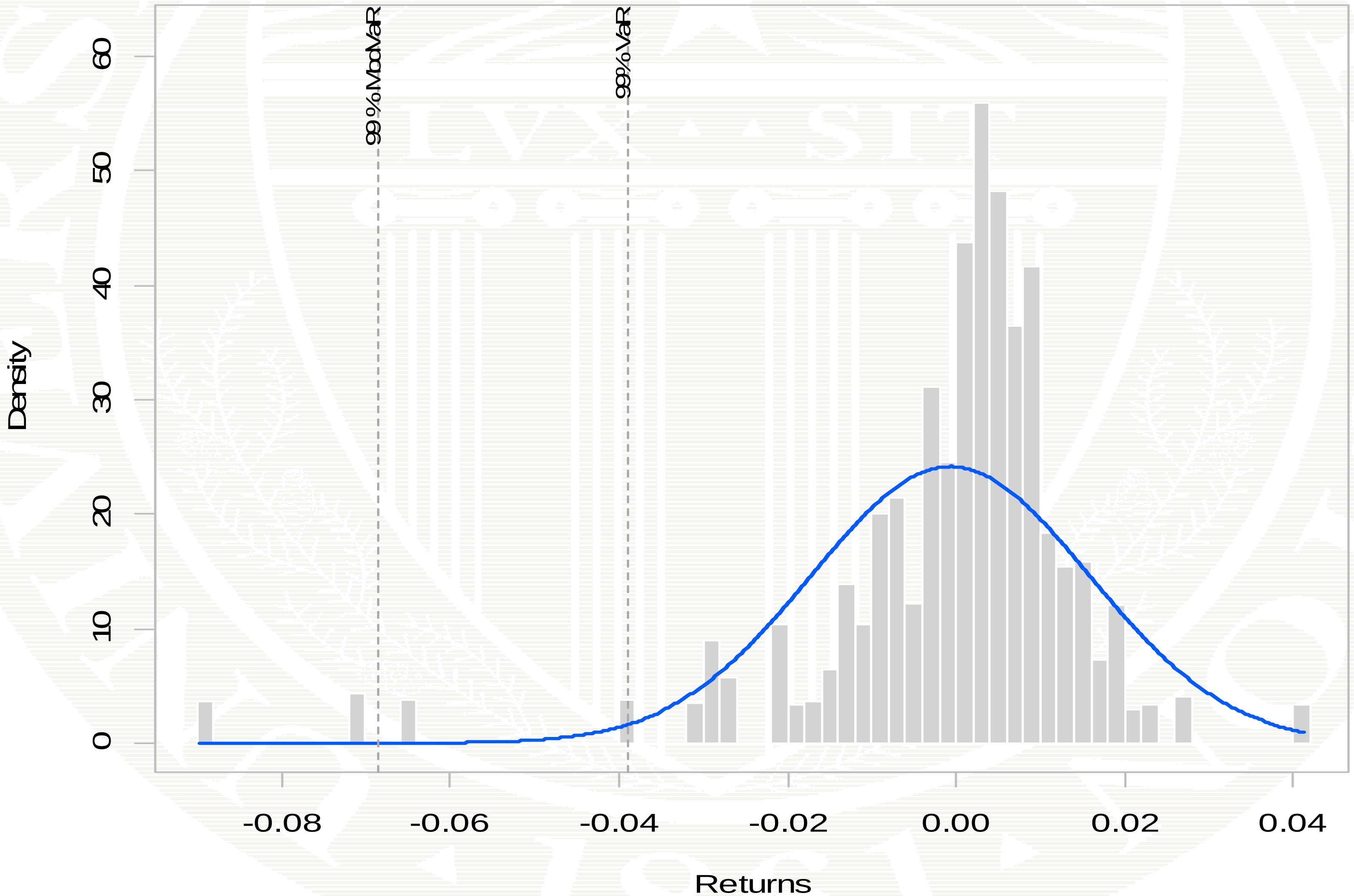
What to do with $\left\{\tilde{R}_{i,u}\right\}_{u=1}^{M}$?

- Backfill missing fund performance
- Compute fund and portfolio performance measures
- Estimate non-parametric fund and portfolio tail risk measures
- Compute non-parametric risk budgeting measures
- Standard errors can be computed using a bootstrap procedure

Example: Backfilled Fund Performance



Example: Simulated portfolio distribution



S-PLUS and S+FinMetrics vs R

- Dealing with time series objects in R can be difficult and confusing
  - timeSeries, zoo, xts
- Time series regression in R is incompletely implemented
  - Diagnostic plots, prediction
- R packages give about 80% functional coverage to S+FinMetrics

Some Thoughts About Using R in a Corporate Environment

- IT doesn't want to support it
- Firewalls block R downloads
- The world runs from an Excel spreadsheet
- Analysts with some programming experience learn R quickly
- Not good for the casual user

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