Can you do better than cap-weighted equity benchmarks?

Guy Yollin
Principal Consultant, r-programming.org
Visiting Lecturer, University of Washington

Krishna Kumar
Financial Consultant

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INVESTING ALWAYS INVOLVES RISK

Outline

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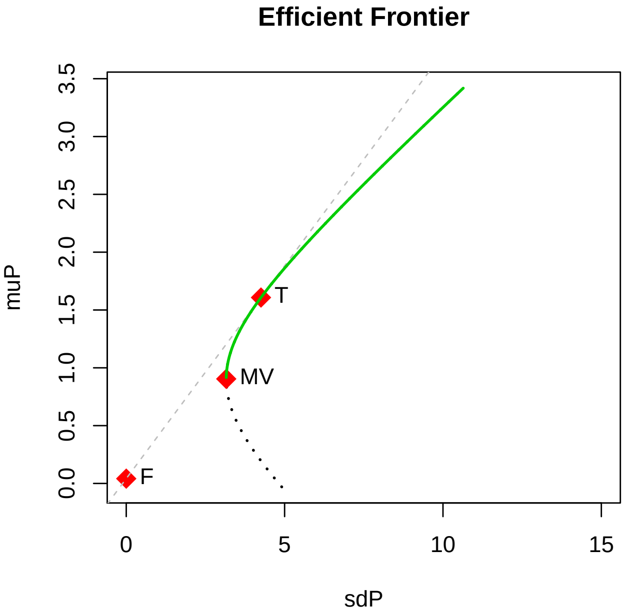
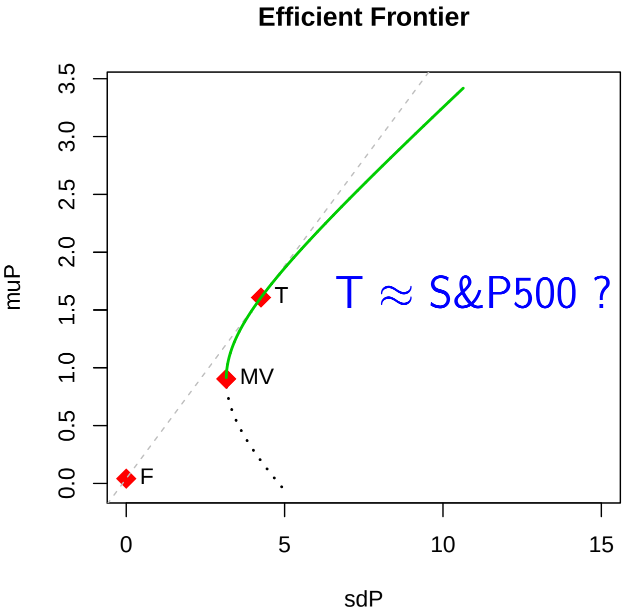
1. Introduction to efficient indexes
2. Overview of modeling
3. Analysis of results
4. Wrap-Up

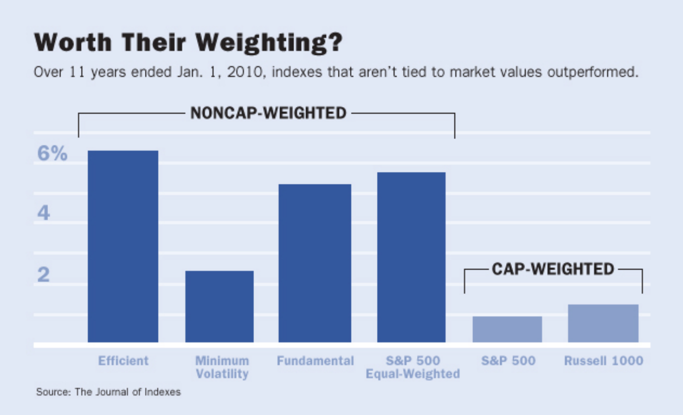
The tangency portfolio



The tangency portfolio



Is Your Index Fund Broken?



Jack Hough, SmartMoney, "Is Your Index Fund Broken?", January 31, 2011

A New Idea?

The efficient market inefficiency of capitalization-weighted stock portfolios

"Matching the market is an inefficient investment strategy."

Robert A. Haugen and Nardin L. Baker

Haugen and Baker, Journal of Portfolio Management, "The efficient market inefficiency of capitalization-weighted stock portfolios", Spring 1991

Efficient Indexation

- maximize Sharpe ratio

$$w^* = \arg\max_w \frac{w'\mu}{\sqrt{w'\Sigma w}}$$

- covariance matrix
  - derived from principal component analysis (PCA)
- expected returns
  - form deciles by downside risk
  - expected return equals mean of each decile


An EDHEC-Risk Institute Publication

Efficient Indexation: An Alternative to Cap-Weighted Indices

January 2010

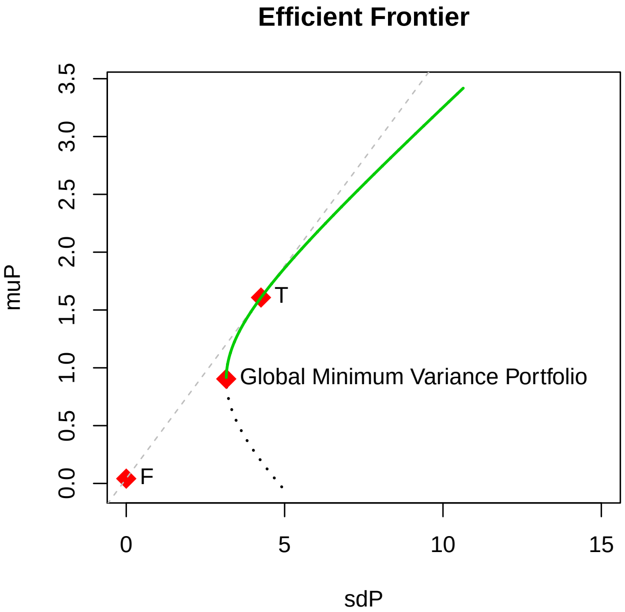
Amenc, Goltz, Martellini, "Efficient Indexation: An Alternative to Cap-Weighted Indices", January 2010

Research project

- Goal
  - Compare performance of alternative index constructions using S&P 500 constituents
- Methodology
  - use a rolling 2-year window of current constituent returns and re-balance at the start of each month
  - generate 48-months of out-of-sample index returns (Jan-2007 to Dec-2010)
  - S&P 500 returns were calculated using constituent weights (apples-to-apples comparisons without factoring in transaction costs)
- Constraint
  - positive weights (max of 25%)
- Focus of research
  - minimum risk (minimum variance and minimum CVaR) portfolios

Outline

Global minimum variance portfolio



M-V optimization and Quadratic Programming

general QP problem

$$min_b \quad \frac{1}{2}\mathbf{b^T D b} - \mathbf{b^T d}$$

$$s.t. \quad \mathbf{A^T b} \geq \mathbf{b_0}$$

$$\mathbf{b} \geq 0$$

mean-variance portfolio optimization

$$min_b \quad \boldsymbol{\omega}^T \boldsymbol{\Sigma} \boldsymbol{\omega}$$

$$s.t. \quad \boldsymbol{\omega}^T \boldsymbol{\mu} = \mu_p$$

$$\boldsymbol{\omega}^T \mathbf{1} = 1$$

$$\omega_{min} \geq \omega_i \geq \omega_{max}$$

R Code: the solve.QP function

```
> library(quadprog)
> args(solve.QP)

function (Dmat, dvec, Amat, bvec, meq = 0, factorized = FALSE)
NULL
```

objective function assignments: $2\boldsymbol{\Sigma} \rightarrow \mathbf{D} \quad \boldsymbol{\omega} \rightarrow \mathbf{b} \quad \mathbf{0} \rightarrow \mathbf{d}$

Factor models for asset returns

The general form of a factor model for asset returns is:

$$R_{j,t} = \beta_{0,j} + \beta_{1,j} F_{1,t} + \cdots + \beta_{p,j} F_{p,t} + \epsilon_{j,t}$$

where

$R_{j,t}$ is either return or excess return on the jth asset at time t

$F_{1,t}, \ldots, F_{p,t}$ are factors (aka risk factors) at time t

$\epsilon_{1,t}, \ldots, \epsilon_{n,t}$ are uncorrelated, mean-zero, unique risks

The factor model in matrix form is:

$$\mathbf{R_t} = \boldsymbol{\beta}_0 + \boldsymbol{\beta}^T \mathbf{F_t} + \boldsymbol{\epsilon}_t$$

Returns covariance matrix

Given the following covariance matrices:

$$\boldsymbol{\Sigma}_\epsilon = \begin{pmatrix} \sigma^2_{\epsilon,1} & \cdots & 0 \\ \vdots & \sigma^2_{\epsilon,j} & \vdots \\ 0 & \cdots & \sigma^2_{\epsilon,n} \end{pmatrix}$$

$$\boldsymbol{\Sigma}_F = p \times p \text{ covariance matrix of } (F_t)$$

The returns covariance matrix is:

$$\boldsymbol{\Sigma}_R = \boldsymbol{\beta}^T \boldsymbol{\Sigma}_F \boldsymbol{\beta} + \boldsymbol{\Sigma}_\epsilon$$

Covariance matrix estimation

- Estimating the covariance matrix based on a factor model is a bias-versus-variance trade-off
  - sample covariance matrix is unbiased but may have *significant* estimation error
  - estimating the covariance matrix via a factor model may be biased but also may significantly reduce estimation error by significantly reducing the number of estimates

- Sample covariance matrix for n-assets
  - $n(n+1)/2$ estimations
  - for 500 assets, 125,250 estimates are required

- Covariance matrix with n-assets and a factor model with p-factors
  - $np + n + p^2$ estimations
  - for 500 assets and 10 factors, 5,600 estimates are required

Industry factor model

Model background

- Sheikh, "Barra's Risk Models", 1995

Response

- daily equity returns

Explanatory variables

- company industry classification

Model details

- Example 103, Zivot, "Modeling Financial Time Series with S-PLUS, 2nd Edition", 2005
  `http://faculty.washington.edu/ezivot/book/Ch15.factorExamples2ndEdition.ssc`

Cross-sectional factor models

Differences between time-series factor models and cross-sectional factor models:

Model type	**Assets**	**Time Periods**	**Factors**	**Betas**
time-series	one asset at a time	all time periods	known	estimated
cross-section	all assets	one period at a time	estimated	known

Cross-sectional factor model for the jth asset at some fixed t:

$$R_j = \beta_0 + \beta_1 F_{1,j} + \cdots + \beta_p F_{p,j} + \epsilon_j$$

Industry factor model

General industry factor model has the following form:

$$R_j = \beta_1 F_{1,j} + \beta_2 F_{2,j} + \cdots + \beta_p F_{p,j} + \epsilon_j$$

$$\beta_i = \begin{cases} 1, & \text{if asset j in industry i} \\ 0, & \text{if asset j not in industry i} \end{cases}$$

- Factor realizations represent a weighted average return in time period t of all of the asset returns for companies operating in industry j
- S&P Sector GICS codes for 10 sectors (10 sectors):

energy	materials	industrial	discretionary	staples
health	financial	info tech	telecom	utilities

Statistical factor models

Recall the general form of a factor model:

$$\mathbf{R_t} = \boldsymbol{\beta}_0 + \boldsymbol{\beta}^T \mathbf{F_t} + \epsilon_t$$

- In statistical factor models:
  - factor realizations are not directly observable
  - no external knowledge of betas (as in cross-sectional models)
  - factor realizations and betas must be extracted from the returns data using statistical methods
- Principal component analysis - eigen decomposition of covariance matrix

PCA statistical factor model

Model background

- "Modeling Financial Time Series with S-PLUS, 2nd Edition", 2005

Response

- daily equity returns

Explanatory variables

- principal components

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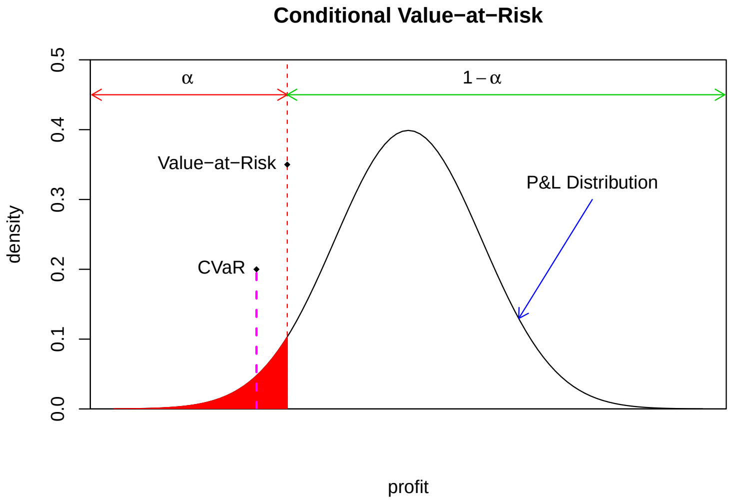
Conditional Value-at-Risk



CVaR Optimization via Linear Programming

It can be shown that minimizing the CVaR of a portfolio is a linear programming problem that can be carried out with a general-purpose LP solver[†]

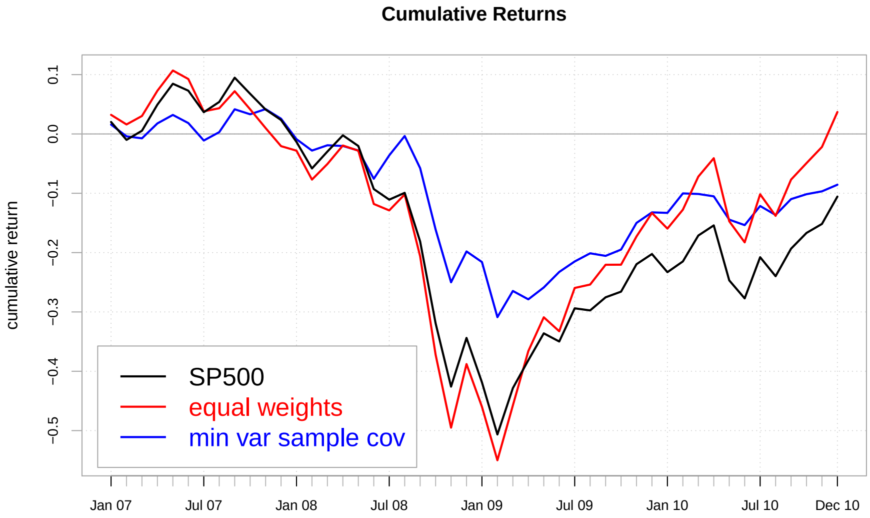
R Code: the Rglpk_solve_LP

```
> library(Rglpk)

Using the GLPK callable library version 4.42

> args(Rglpk_solve_LP)

function (obj, mat, dir, rhs, types = NULL, max = FALSE, bounds = NULL,
    verbose = FALSE)
NULL
```

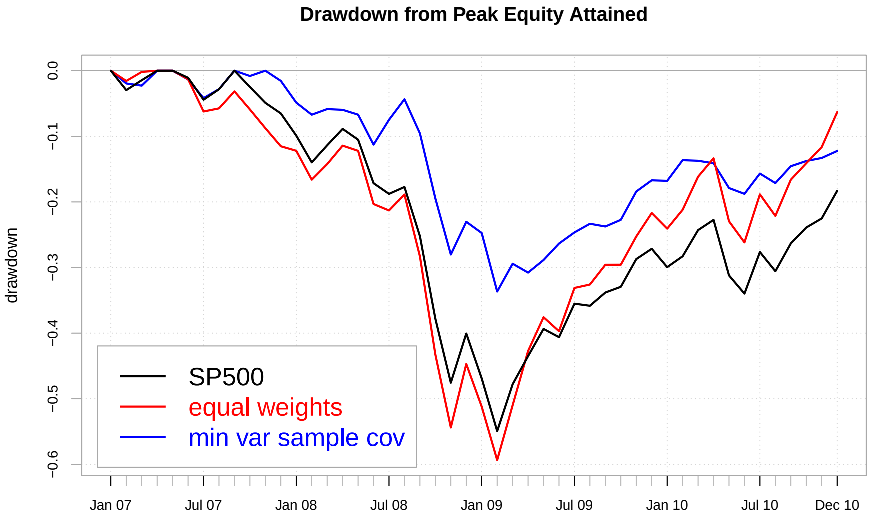
[†]Yollin, "R Tools for Portfolio Optimization", R/Finance 2009

Outline

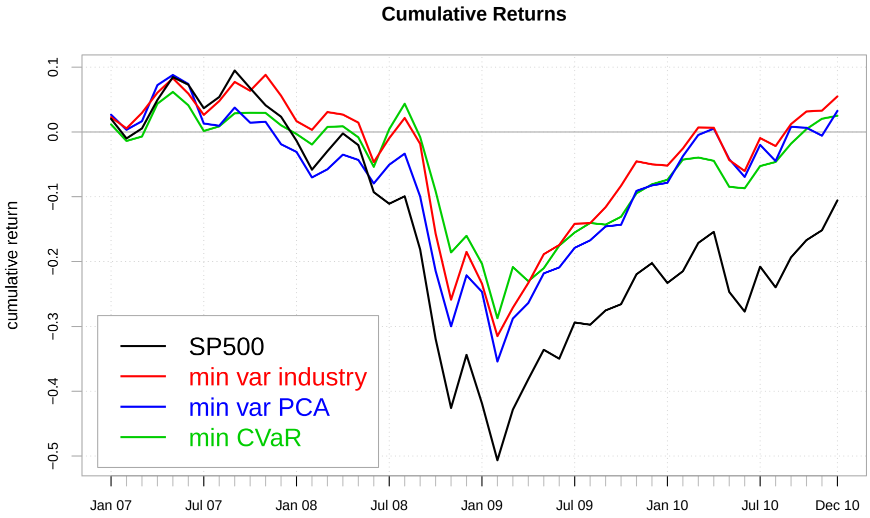
Cumulative return comparisons



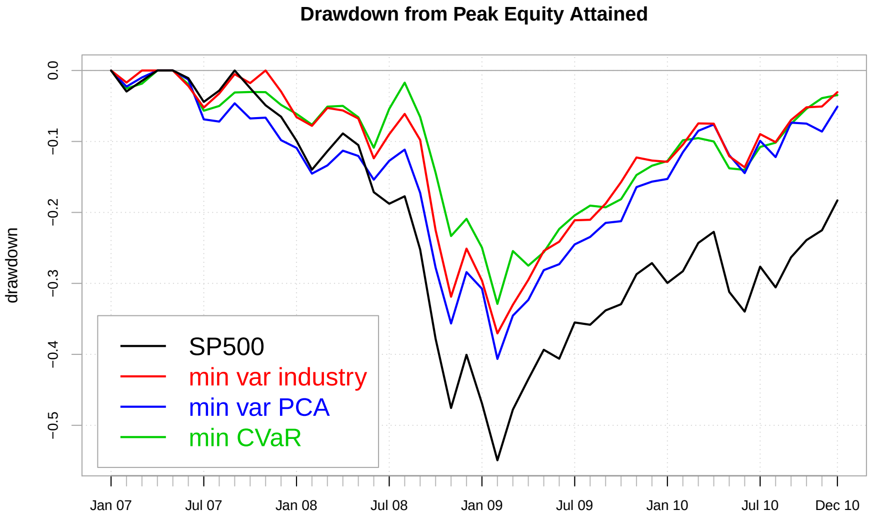
Cumulative return comparisons



Cumulative return comparisons



Cumulative return comparisons



Summary

	SP500	minVaRSample	minVarIndustry	minVarPCA	minCVaR
Cumulative Return	-0.106	-0.086	0.055	0.032	0.025
Annualized Return	-0.028	-0.022	0.013	0.008	0.006
Annualized StdDev	0.241	0.138	0.161	0.174	0.139
Conditional VaR	-0.159	-0.105	-0.118	-0.126	-0.100
Max DrawDown	0.549	0.337	0.370	0.406	0.329

- all minimum variance portfolios and the minimum CVaR portfolio outperformed the S&P 500 Index during the testing period
  - higher annualized return
  - lower annualized volatility
  - smaller conditional value-at-risk
  - smaller maximum drawdown
- returns are difficult (impossible) to forecast and these techniques don't require them

Can you do better than cap-weighted equity benchmarks? Maybe!

Outline

Special thanks

SunGard Financial Systems

- Historical S&P 500 constituent weights
- Historical stock prices

Special thanks

Revolution Analytics

- Revolution R Enterprise and RevoScaleR

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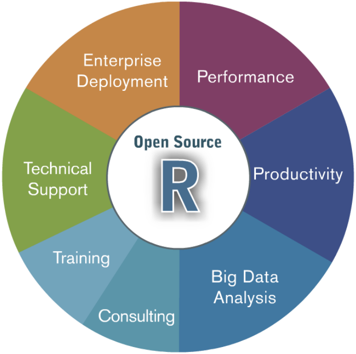

Q & A

- Questions and comments
- Contacting the Presenters
  - Guy Yollin
    - `http://www.r-programming.org`
    - gyollin@r-programming.org
  - Krishna Kumar
    - kk2250@gmail.com